Scale Invariance without Inflation?

C. Armendáriz-Picón
Enrico Fermi Institute and Department of Astronomy and Astrophysics, University of Chicago.

Eugene A. Lim
Center for Cosmological Physics and Department of Astronomy and Astrophysics, University of Chicago.

We propose a new alternative mechanism to seed a scale invariant spectrum of primordial density perturbations that does not rely on inflation. In our scenario, a perfect fluid dominates the early stages of an expanding, non-inflating universe. Because the speed of sound of the fluid decays, perturbations are left frozen behind the sound horizon, with a spectral index that depends on the fluid equation of state. We explore here a toy model that realizes this idea. Although the model can explain an adiabatic, Gaussian, scale invariant spectrum of primordial perturbations, it turns out that in its simplest form it cannot account for the observed amplitude of the primordial density perturbations.

Keywords: cot, inf

I. INTRODUCTION

During the last decade, several experiments have probed the nature of the primeval perturbations that gave rise to the anisotropies observed in the universe [1]. The results of these measurements are consistent with an adiabatic, Gaussian and nearly scale invariant primordial spectrum of perturbations, as predicted by the simplest models of inflation [2]. Due to the central role that the origin of these perturbations plays in our understanding of the early universe, it is important to ascertain to what extent alternatives to seed such a spectrum exist. In the context of a universe dominated by a single canonical scalar field coupled to general relativity, this issue has been investigated in [3]. In a contracting universe, there are two ways to seed a scale invariant spectrum [4], and in an expanding universes only a stage of de Sitter inflation can generate it. Other alternatives that relax some of the assumptions made in [3] have been discussed in [5, 6, 7, 8]. It is fair to say though, that the only presently known, widely accepted way to seed a scale invariant spectrum of perturbations in an expanding universe requires a stage of de Sitter inflation.

During inflation, the physical length of a perturbation grows faster than the Hubble radius H^{-1}. Hence, because for a scalar field the sound of speed is one, initially sub-horizon sized modes cross the *sound* horizon and subsequently freeze. Note that the crossing of the sound horizon $c_s H^{-1}$, rather than the Hubble radius, is what leads to the causal seeding of a primordial spectrum of density perturbations.

In this paper we propose an alternative seeding mechanism in an *expanding* universe that does not require inflation. In our scenario, modes cross the sound horizon because the sound speed decreases sufficiently fast. The amplitude of the perturbations depends on the values of the Hubble parameter and the sound speed at the time of crossing. If the sound speed decays appropriately, it is also possible to seed a scale invariant spectrum. In order to push the seeded perturbations to super-Hubble scales, it is necessary that a period of inflation follows the seeding. However, let us emphasize that the seeding and inflation are two distinct, unrelated cosmological phases; the nature of the seeded spectrum is independent of the properties of the inflationary stage, which does not have to be de Sitter-like.

We discuss a particular realization of the scenario outlined above. Although the particular model we discuss can successfully seed a scale invariant spectrum of density perturbations, it turns out that it cannot account for the observed amplitude. Specifically, if one requires the amplitude of the seeded perturbations to agree with the one observed in CMB anisotropies, our model can seed only about two decades in k space (instead of the required three to four). On the other hand, if one requires the seeding to span about three decades, then the amplitude of the seeded perturbations has to be smaller than the observed one. The only way to escape this conclusion is to assume that between seeding and observation the amplitude of the seeded spectrum is boosted to the observed value by a different mechanism.

II. FORMALISM

Consider a spatially flat FRW universe dominated by an isentropic perfect fluid. We would like to study scalar perturbations in such a universe [9], so we shall deal with the perturbed metric (in longitudinal gauge)

$$ds^2 = a^2(\eta)\left[(1+2\Phi)d\eta^2 - (1-2\Phi)d\vec{x}^2\right]. \qquad (1)$$

The evolution of the scale factor is determined by the background Einstein equations

$$\mathcal{H}^2 = a^2\rho \quad \text{and} \quad \mathcal{H}' = -\frac{\mathcal{H}^2}{2}(1+3w), \qquad (2)$$

where $\mathcal{H} = a'/a$ and prime denotes a derivative with respect to conformal time η. We work in units where Newton's constant is $G = 3/(8\pi)$ and, as usual, we have defined the equation of state parameter $w = p/\rho$. The pressure of the fluid is p, and ρ is its energy density. The Hubble parameter is $H = \mathcal{H}/a$.

The evolution of the scalar perturbations can be described in terms of the Mukhanov variable v, which is a linear combination of the gravitational potential Φ and the fluid perturbations [9, 10]. The Fourier components of the Mukhanov variable v_k obey the simple differential equation

$$v_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0, \tag{3}$$

where the variable z is given by

$$z = \frac{a(1+w)^{1/2}}{c_s}, \tag{4}$$

and the speed of sound of the isentropic fluid is

$$c_s^2 \equiv \frac{dp}{d\rho} = w - \frac{w'}{3\mathcal{H}(1+w)}. \tag{5}$$

Note that for isentropic fluids, the equation of state and the speed of sound are linked by the fluid equation of motion $\rho' + 3\mathcal{H}\rho(1+w) = 0$.

Whereas the variable v proves to be useful to study the quantization and the evolution of the perturbations, the Bardeen variable

$$\zeta = \frac{v}{z} = \frac{2}{3}\frac{\mathcal{H}^{-1}\Phi' + \Phi}{1+w} + \Phi \tag{6}$$

turns to be more convenient to compute observable quantities, such as the temperature anisotropies in the CMB radiation. These anisotropies can be characterized by the the power spectrum

$$\mathcal{P}_\zeta = \frac{k^3}{4\pi^2}|\,\zeta_k|^2, \tag{7}$$

which is a measure of the mean square fluctuations of the variable ζ on comoving scales $1/k$. The variable ζ is also useful because it can be directly related to the perturbations in the energy density. In fact, in cases where Φ is constant, it follows from Eq. (6) that $\zeta_k \sim \Phi_k$. Using Poisson's equation (for modes smaller than the Hubble radius) we then find

$$\mathcal{P}_{\delta\rho/\rho} \sim \frac{k^4\,\mathcal{P}_\zeta}{a^4\,\rho^2}, \tag{8}$$

which connects the inhomogeneities in ζ to the inhomogeneities in the fractional density contrast.

Let us turn our attention to the equation of motion of the perturbations, Eq. (3). By definition, in the short-wavelength regime $c_s^2 k^2 \gg z''/z$. If c_s and w are constant, the last condition translates into $c_s^2 k^2 \gg a^2 H^2$, which means that modes are smaller than the sound horizon. In this regime, an approximate solution of Eq. (3) is

$$v_k(\eta) = \frac{1}{\sqrt{c_s(\eta)k}} \exp\left(-i\int^\eta c_s(\tilde{\eta})\,k\,d\tilde{\eta}\right). \tag{9}$$

This WKB-like solution is valid as long as the change in the frequency $c_s k$ is adiabatic,

$$c_s^2 k^2 \gg \frac{c_s''}{c_s} \quad \text{and} \quad c_s^2 k^2 \gg \left(\frac{c_s'}{c_s}\right)^2. \tag{10}$$

Then, the approximate solution (9) corresponds to zeroth-order adiabatic vacuum initial conditions [11]. In the long-wavelength regime, $c_s^2 k^2 \ll z''/z$, one of the solutions of Eq. (3) is proportional to z. Hence, if $v \propto z$ is the dominant solution of Eq. (3), for long-wavelength modes the variable ζ is conserved. This remains true as long as there are no entropy perturbations.

III. DECAYING SOUND SPEED

In order to causally seed a spectrum of perturbations it is necessary that modes that initially lie in the short-wavelength regime, where natural initial conditions exist, enter the long-wavelength regime and subsequently freeze. This occurs only if

$$\left(\frac{z''}{c_s^2 z}\right)' > 0. \tag{11}$$

One way to satisfy condition (11) is to assume that $c_s = 1$ and $w < -1/3$. Then, $z''/z \sim a^2 H^2$ increases while c_s remains constant. In physical terms, this means the physical length of a mode, a/k, grows faster than the Hubble radius H^{-1}, the condition that singles out inflation. Recall that for a (canonical) scalar field c_s is indeed equal one [10].

There are nevertheless additional ways to satisfy the seeding condition (11). The main idea of our paper is that a rapidly changing speed of sound can also result in the seeding of a scale invariant spectrum. Even if $z''/z \sim a^2 H^2$ decreases in time, if c_s changes fast enough it is possible to satisfy condition (11). Physically, this means that the length of a mode grows faster than the *sound* horizon $c_s H^{-1}$. In the following we present a model that realizes this idea. For simplicity, we assume here that the fluid that dominates the universe is isentropic. In that case, the sound speed is determined by the equation of state, which in turn is determined by the expansion of the universe. Hence, the free parameters of our simple model are tightly constrained. By considering non-isentropic fluids or non-canonical scalar fields, one could avoid these restrictions.

IV. A SIMPLE MODEL

Suppose that the early universe is dominated by an isentropic fluid with polytropic equation of state

$$p = w_* \rho_* \left(\frac{\rho}{\rho_*}\right)^{1+\alpha/3}. \tag{12}$$

Here α is a free parameter that characterizes the polytropic index and w_* is the value of the equation of state parameter for $\rho = \rho_*$. Perfect fluids with polytropic equations of state have been long and widely considered in astrophysics [12].

If $w_* \ll 1$, for energy densities below ρ_* the pressure p is negligible. Under the assumption $p \ll \rho$, integration of Eqs. (2) yields

$$a = a_* \left(\frac{\eta}{\eta_*}\right)^2, \quad \mathcal{H} = \frac{2}{\eta}, \tag{13}$$

which is of course how a dust-dominated universe evolves in conformal time. Although the equation of state parameter and the sound speed are small, they are not exactly zero. Using Eqs. (12), (2) and (5) we find

$$w = w_* \left(\frac{\eta}{\eta_*}\right)^{-2\alpha}, \quad c_s^2 = \left(1 + \frac{\alpha}{3}\right) w(\eta). \tag{14}$$

Finally, substituting these expressions into Eq. (4) we arrive at

$$\frac{z''}{z} = \frac{2 + 3\alpha + \alpha^2}{4} \mathcal{H}^2. \tag{15}$$

Therefore, the seeding condition Eq. (11) is satisfied only if c_s decays fast enough, $\alpha > 1$. The origin of this condition can be also interpreted in physical terms. The physical length of a mode is $\lambda = a/k$, and the size of the sound horizon is $c_s H^{-1}$. Requiring the physical length of a mode to grow faster than the sound horizon also yields $\alpha > 1$. Note that if $\alpha > 3$, both speed of sound and sound horizon decay.

With z''/z given by Eq. (15) and c_s^2 given by Eq. (14) the differential equation (3) has the solution

$$v_k = \eta^{1/2} \left[A_k H_\nu \left(\frac{c_s\, k}{\alpha - 1}\eta\right) + B_k H_\nu^* \left(\frac{c_s\, k}{\alpha - 1}\eta\right)\right], \tag{16}$$

where H_ν is the Hankel function of the first kind and

$$\nu = \frac{2\alpha + 3}{2(\alpha - 1)}. \tag{17}$$

The coefficients A_k and B_k are integration constants determined by the initial conditions. For modes well within the short-wavelength regime, the adiabaticity conditions (10) are satisfied. Setting

$$A_k = \sqrt{\frac{\pi}{2(\alpha - 1)}}, \quad B_k = 0 \tag{18}$$

the solution (16) approaches the adiabatic vacuum (9). Substituting Eq. (16) into Eq. (7) we find that in the long-wavelength regime the power-spectrum $\mathcal{P}_\zeta$ is time-independent. Expressing it in terms of the values of H and c_s at the time a reference scale k_* crosses the sound horizon, $c_s^* k_* = a_* H_*$, results into

$$\mathcal{P}_\zeta = \frac{\Gamma^2(\nu) \cdot (\alpha - 1)^{2\nu - 1}}{4\pi^3} \cdot \frac{H_*^2}{c_s^*} \left(\frac{k}{k_*}\right)^{n_s - 1}, \tag{19}$$

where ν is determined by Eq. (17) and the spectral index is given by

$$n_s - 1 = \frac{\alpha - 6}{\alpha - 1}. \tag{20}$$

Hence, for $\alpha > 6$ the spectrum is blue, for $\alpha = 6$ it is scale invariant[1] and for $\alpha < 6$ it is red. In particular, one can imagine a situation where α is mildly ρ-dependent and increasing as a function of ρ. In such a case, the spectral index would run from $n_s > 1$ to $n_s < 1$ at smaller scales, as hinted by recent observations [1]. Because the seeded perturbations originate from quantum fluctuations of the fluid, they are expected to be Gaussian.

A. Observational constraints

Large scale structure and CMB measurements probe the primordial spectrum on scales that range from the size of today's Hubble radius to scales about 10^3 times smaller. Presently, these probes are consistent with an adiabatic, Gaussian, nearly scale invariant spectrum with amplitude $\mathcal{P}_\zeta \sim 10^{-10}$ [1]. In this section we explore whether our model is capable of successfully accounting for these primordial anisotropies and how these observations constrain and affect our model.

Suppose that the seeding of perturbations we have described lasts for ΔN_s e-folds. The minimal value of ΔN_s is determined by the requirement that our mechanism seeds perturbations spanning three decades in k space. Because modes are seeded when they cross the sound horizon, $c_s k = aH$, using Eqs. (13) and (14) one thus finds that the number of e-folds of seeding has to satisfy

$$\Delta N_s \geq \frac{D \log 10}{1 + \alpha/2 - 3/2}, \tag{21}$$

where D is the number of cosmologically relevant decades in k space (about three). After the seeding, we assume that the universe evolves with constant equation of state $\widetilde{w}$ for a period of $\Delta N_{\widetilde{w}}$ e-folds. Next, the universe is reheated at a_{rh} and it subsequently undergoes a radiation dominated phase until equipartition a_{eq}. Following

[1] Although this statement is true in our approximation $w \to 0$, the arbitrarily small deviations from $w = 0$ will cause deviations from scale invariance even if $\alpha = 6$.

equipartition the universe becomes matter dominated until today a_0 (for simplicity we neglect the recent stage of cosmic acceleration). Thus, at the beginning of the seeding, the physical size of a comoving scale k_* was

$$\lambda_* = \frac{a_0}{k_*}\frac{a_{eq}}{a_0}\frac{a_{rh}}{a_{eq}}\exp(-\Delta N_{\widetilde{w}})\exp(-\Delta N_s). \tag{22}$$

On the other hand, the value of the Hubble parameter at that time was

$$H_* = H_0\left(\frac{a_0}{a_{eq}}\right)^{3/2}\left(\frac{a_{eq}}{a_{rh}}\right)^2 e^{3\Delta N_{\widetilde{w}}(1+\widetilde{w})/2}e^{3\Delta N_s/2}. \tag{23}$$

Using the last two equations, one can compute the value of the speed of sound at the time the present Hubble radius $k_* = a_0 H_0$ crossed the sound horizon during seeding, $c_s^* H_*^{-1} = \lambda_*$,

$$c_s^* = \left(\frac{a_0}{a_{eq}}\right)^{1/2}\left(\frac{a_{eq}}{a_{rh}}\right)e^{(1+3\widetilde{w})\Delta N_{\widetilde{w}}/2}e^{\Delta N_s/2}. \tag{24}$$

Equipartition occurred at $a_0/a_{eq} \approx 3\cdot 10^3$, and because reheating has to happen before nucleosynthesis, $a_{eq}/a_{rh} \geq 10^8$. Since the speed of sound and the equation of state parameter are related by Eq. (14), inspection of Eq. (24) shows that in order to prevent a violation of the condition $w \ll 1$, we have to assume that the seeding was followed by a stage of inflation, $\widetilde{w} < -1/3$.

The evolution of a particular comoving scale k in time is depicted in Fig. 1. Observe that in our model, the seeding of the scalar perturbations and the inflationary epoch are decoupled from each other. The only purpose of the inflationary stage is to allow for a small speed of sound during the seeding and to push the already seeded spectrum of perturbations to scales larger than the Hubble radius. Gravitational waves are seeded only during the stage of inflation. Because the amplitude of the tensor perturbations at the time a given scale crosses the Hubble radius is proportional to H, the tensor to scalar ratio at $k_* = a_0 H_0$ is, up to constants of order one,

$$\left.\frac{\mathcal{P}_h}{\mathcal{P}_\zeta}\right|_{k_*} \sim (c_s^*)^{-\frac{5+3\widetilde{w}}{1+3\widetilde{w}}}\cdot\exp\left(-\frac{6\widetilde{w}}{1+3\widetilde{w}}\Delta N_s\right). \tag{25}$$

Since c_s^* is small, tensor perturbations are suppressed. The same argument applies to scalar perturbations generated during any inflationary regime driven by a component different from the seeding fluid.

Inflation might be driven by the same component that causes the seeding, e.g. by a k-field [13], or it might be driven by a second component, like a conventional scalar. In the former case, in order to preserve the seeded spectrum, it is important that perturbations do not reenter the sound horizon during inflation. This is possible during k-inflation, because the speed of sound and the equation of state parameters are two arbitrary independent quantities. In the latter case, it is important that no entropic component be generated during inflation. Just as

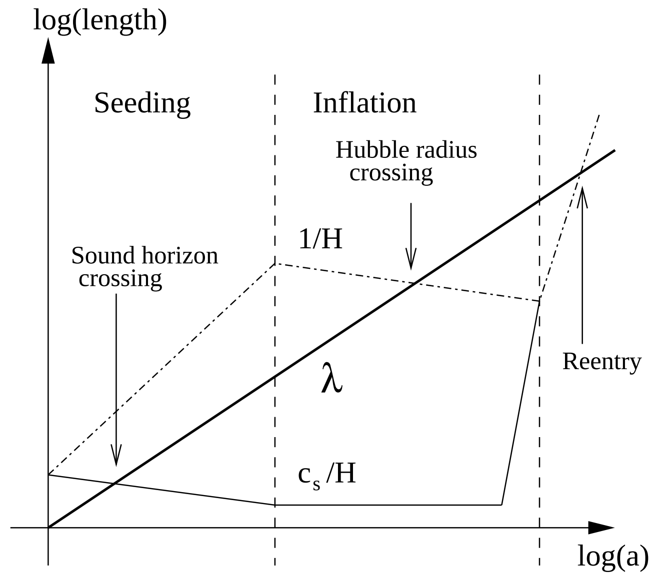

FIG. 1: A schematic plot of the evolution of the different length scales in our scenario. During seeding, the physical length λ (thick line) crosses the sound horizon c_s/H (continuous line). Later, during inflation, λ exits the Hubble radius $1/H$ (dot-dashed line).

for gravitational waves, such entropic perturbations are expected to be negligible. Nevertheless, the presence of a second component could have non-trivial consequences, such as for instance, a boost in the primordial amplitude of the perturbations due to parametric resonance during reheating [14].

It turns out that the validity of perturbation theory severely constrains our model. Using Eq. (8) one can compute the value of $\delta\rho/\rho$ at the time of sound horizon crossing for the last seeded mode[2],

$$\mathcal{P}_{\delta\rho/\rho} = \frac{\mathcal{P}_\zeta}{{c_s^*}^4\exp\left(-2\alpha\Delta N_s\right)}. \tag{26}$$

The validity of perturbation theory requires $\mathcal{P}_{\delta\rho/\rho} < 1$, and self-consistency of our derivation $c_s^* < 1$. Therefore we find

$$\Delta N_s < \frac{\log\mathcal{P}_\zeta^{-1}}{2\alpha}. \tag{27}$$

The amplitude of the primordial fluctuations observed in CMB temperature anisotropies implies $\mathcal{P}_\zeta \sim 10^{-10}$ [1]. Setting $\alpha = 6$ ($n_s = 1$) in the last equation we thus find $\Delta N_s < 2$, which is somewhat smaller than the observationally required $\Delta N_s \approx 3$ that follow from Eq. (21) (for $D = 3$).

[2] The constraint that follows by considering this mode is stronger than the one requiring that the density perturbation for the mode k_* remain in the linear regime.

B. A Microscopic Description

Although Eq. (12) uniquely specifies the perfect fluid model, it might be desirable to cast our model into a microscopic formulation. Consider a scalar field φ with Lagrangian

$$L = p(X), \quad \text{where} \quad X = \frac{1}{2} g^{\mu\nu} \frac{\partial \varphi}{\partial x^\mu} \frac{\partial \varphi}{\partial x^\nu}. \tag{28}$$

Here, p is an as of yet unspecified, arbitrary function of X. Such a Lagrangian describes a k-field [13]. For time-like field gradients, the energy momentum tensor of the k-field has perfect fluid form, with pressure given by $p(X)$ and energy density given by $\rho(X) = 2X dp/dX - p$. Because the Lagrangian only depends on X, in this particular case the k-field behaves as an isentropic fluid. Using the previous expression for $\rho(X)$ one can derive a differential equation for the $p(X)$ that leads to the desired equation of state (12). The solution is

$$p(X) = w_* \rho_* \left[1 + \frac{\alpha}{2w_*(3+\alpha)} \log\left(\frac{X}{X_*}\right) \right]^{1+3/\alpha}, \tag{29}$$

where $p \ll \rho$ has been assumed.

V. CONCLUSIONS

We have presented a novel alternative mechanism to causally seed a spectrum of density perturbations in an expanding universe. In our scenario, the decaying sound speed of an isentropic perfect fluid causes perturbations to be left frozen behind the sound horizon, with a spectrum that depends on the decay rate of the sound speed. In order for the seeded spectrum to correspond to cosmologically relevant scales, an inflationary stage has to follow the phase of perturbation seeding. Therefore, the seeding and the spectrum of the perturbations and the inflationary stage are distinct unrelated cosmological phases. Since the inflationary stage happens after the seeding, the gravitational waves generated during the inflationary stage are suppressed compared to scalar perturbations.

Although the particular model we have discussed can successfully explain a scale invariant spectrum of Gaussian, adiabatic perturbations on observable scales, it cannot account for its observed amplitude in the cosmologically relevant window of about three decades in k space. The only way we are aware of to avoid this conclusion is to assume that parametric resonance during reheating boosts the amplitude of the primordial spectrum to its observed value [14]. At this point one should also bear in mind that we have explored only one particular realization of our scenario. In fact, we are not aware of any physical principle that would prevent other realizations to also account for the correct amplitude in a larger cosmological window.

Why should we consider a stage of pre-inflationary seeding if perturbations can be generated during inflation anyway? It turns out that in some cases it might be useful to decouple the seeding of the perturbations from the stage of inflation [15]. This decoupling acquires particular relevance in the light of the recent signs of a running spectral index [1]. Such a running can be accommodated by conventional inflationary models only at the cost of spoiling their simplicity and economy [16]. If in the era of precision cosmology one is forced to replace simplicity by phenomenological success, less simple alternatives to the conventional models might be attractive too.

On the other hand, our results can be also interpreted from a different perspective: They display how many hurdles one has to overcome in order to seed a phenomenologically realistic spectrum of perturbations without inflation.

Acknowledgments

We thank R. Brandenberger, Sean Carroll, Wayne Hu and V. Mukhanov for useful comments and discussions. CAP was supported by the US DOE grant DE-FG02-90ER40560, and EAL was supported by the US DOE grant DE-FG02-90ER40560 and the Packard foundation.

[1] D. N. Spergel *et al.*, “First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters,” arXiv:astro-ph/0302209.

[2] V. F. Mukhanov and G. V. Chibisov, “Quantum Fluctuation And Nonsingular Universe,” JETP Lett. **33**, 532 (1981).

[3] S. Gratton, J. Khoury, P. J. Steinhardt and N. Turok, “Conditions for generating scale-invariant density perturbations,” arXiv:astro-ph/0301395.

[4] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, “Density perturbations in the ekpyrotic scenario,” Phys. Rev. D **66**, 046005 (2002). [arXiv:hep-th/0109050]. F. Finelli and R. Brandenberger, “On the generation of a scale-invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase,” Phys. Rev. D **65**, 103522 (2002) [arXiv:hep-th/0112249].

[5] R. Brandenberger and P. M. Ho, “Noncommutative spacetime, stringy spacetime uncertainty principle, and density fluctuations,” Phys. Rev. D **66**, 023517 (2002) [AAPPS Bull. **12N1**, 10 (2002)] [arXiv:hep-th/0203119].

[6] S. Hollands and R. M. Wald, “An alternative

to inflation," Gen. Rel. Grav. **34**, 2043 (2002) [arXiv:gr-qc/0205058]. L. Kofman, A. Linde and V. Mukhanov, "Inflationary theory and alternative cosmology," JHEP **0210**, 057 (2002) [arXiv:hep-th/0206088].

[7] P. P. Avelino and C. J. Martins, "Primordial adiabatic fluctuations from cosmic defects," Phys. Rev. Lett. **85**, 1370 (2000) [arXiv:astro-ph/0002413].

[8] D. H. Oaknin, "Spatial correlations of energy density fluctuations in the fundamental state of the universe," arXiv:hep-th/0305068.

[9] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, "Theory Of Cosmological Perturbations," Phys. Rept. **215**, 203 (1992).

[10] J. Garriga and V. F. Mukhanov, "Perturbations in k-inflation," Phys. Lett. B **458**, 219 (1999) [arXiv:hep-th/9904176].

[11] N. D. Birrell and P. C. Davies, "Quantum Fields In Curved Space," (Cambridge University Press, Cambridge, UK, 1982).

[12] S. Chandrasekhar, "An Introduction to the Study of Stellar Structure," (University of Chicago Press, Chicago, USA, 1939).

[13] C. Armendariz-Picon, T. Damour and V. Mukhanov, "k-inflation," Phys. Lett. B **458**, 209 (1999) [arXiv:hep-th/9904075].

[14] F. Finelli and R. H. Brandenberger, "Parametric amplification of metric fluctuations during reheating in two field models," Phys. Rev. D **62**, 083502 (2000) [arXiv:hep-ph/0003172].

[15] D. H. Lyth and D. Wands, "Generating the curvature perturbation without an inflaton," Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002].

[16] D. J. Chung, G. Shiu and M. Trodden, "Running of the scalar spectral index from inflationary models," arXiv:astro-ph/0305193.